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Convection heat transfer from tube banks in crossflow: Analytical approach

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Abstract

The main objective of this analytical study is to investigate heat transfer from tube banks in crossflow under isothermal boundary condition. For this purpose, a control volume is selected from the fourth row of a tube as a typical cell to study the heat transfer from an in-line or staggered arrangement. An integral method of boundary layer analysis is employed to derive closed form expressions for the calculation of average heat transfer from the tubes of a bank, that can be used for a wide range of parameters including longitudinal pitch, transverse pitch, Reynolds and Prandtl numbers. The models for in-line and staggered arrangements are applicable for use over a wide range of parameters when determining heat transfer from tube banks.

Keywords: Forced convection; Heat transfer; Tube bank; Integral method; Boundary layer analysis

1. Introduction

Heat transfer in flow across a bank of tubes is of particular importance in the design of heat exchangers. Heat exchangers are found in numerous industrial applications, such as steam generation in a boiler or air cooling in the coil of an air conditioner. Tube banks are usually arranged in an in-line or staggered manner and are characterized by the dimensionless transverse, longitudinal, and diagonal pitches. Typically, one fluid moves over the tubes, while the other fluid, at a different temperature, passes through the tubes. In this study, the authors are specifically interested in the convection heat transfer associated with crossflow over the tubes.

This study is one of the first attempts to develop analytical models for the heat transfer from tube banks (in-line and staggered). These models are developed in terms of longitudinal and transverse pitches, Reynolds and Prandtl numbers ($\geqslant 0.71$). In developing these models, it is assumed that the flow is steady, laminar, and fully developed.

1.1. Literature review

Based on the pertinent data available up to 1933, Colburn [1] proposed a simple correlation for heat transfer for flow across banks of staggered tubes as follows:

$$Nu_D = 0.33 Re_D^{0.6} Pr^{1/3} \tag{1}$$

This correlation works well for 10 or more rows of tubes in a staggered arrangement and for $10 < Re_D < 40{,}000$. Then Huge [2], Pierson [3], Omohundro et al. [4], Bergelin et al. [5–7], Jones and Monroe [8], Gram et al. [9], Žukauskas [10], Aiba et al. [11,12], and Žukauskas and Ulinskas [13] reported extensive experimental data for heat transfer and fluid friction during viscous flow across in-line and staggered banks of tubes under two thermal boundary conditions. Grimison [14] correlated the experimental data of Huge [2] and Pierson [3] for both arrangements and gave a correlation of the form:

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Nomenclature

c_p	specific heat of fluid [J/kg K]
CV	control volume
D	tube diameter [m]
F_a	arrangement factor
k	thermal conductivity [W/m K]
h	average heat transfer coefficient [W/m^2 K]
L	tube length, m
N	total number of tubes in bank $\equiv N_T N_L$
N_L	number of tubes in the longitudinal direction
N_T	number of tubes in the transverse direction
Nu_D	Nusselt number based on tube diameter $\equiv Dh/k_f$
Pr	Prandtl number $\equiv \nu/\alpha$
Q	total heat transfer rate [W]
Re_D	Reynolds number based on tube diameter $\equiv DU_{max}/\nu$
S_D	diagonal pitch [m]
S_L	longitudinal distance between two consecutive tubes [m]
S_T	transverse distance between two consecutive tubes [m]
$\mathscr{S}_L$	dimensionless longitudinal pitch $\equiv S_L/D$
$\mathscr{S}_T$	dimensionless transverse pitch $\equiv S_T/D$
s	distance along curved surface of tube measured from forward stagnation point [m]
T	temperature [°C]
U_{app}	approach velocity [m/s]
U_{max}	maximum velocity in minimum flow area [m/s]
$U(s)$	velocity in inviscid region just outside boundary layer [m/s]
u	s-component of velocity in boundary layer [m/s]
v	η-component of velocity in boundary layer [m/s]

Greek symbols

α	thermal diffusivity [m^2/s]
ΔT_{lm}	log mean temperature difference [°C]
δ_T	thermal boundary layer thickness [m]
η	distance normal to and measured from surface of tube [m]
λ	pressure gradient parameter
θ	angle measured from stagnation point, radians

Subscripts

a	ambient
f	fluid
o	outlet
p	pressure
T	thermal
w	wall

$$Nu_D = CRe_D^n \tag{2}$$

His empirical relation is valid for air only and pertains to tube banks having 10 or more rows of tubes in the flow direction. For fewer rows, Kays and London [15] gave a correction factor C_2 such that:

$$Nu_D|_{N_L<10} = C_2 Nu_D|_{N_L\geqslant 10} \tag{3}$$

The values of C, C_2 and n are given, in tabular form, in most heat transfer text books (e.g., [16,17]) for both in-line and staggered arrangements. Grimison [14] also correlated the test measurements of Pierson [3] and Huge [2] using a second method and derived the following expression:

$$Nu_D = 0.32F_a Re_D^{0.61} Pr^{0.31} \tag{4}$$

where the arrangement factor F_a was presented graphically by Grimison [14] for the various values of Re_D, subject to the dimensionless transverse and longitudinal pitches.

Hausen [18] modified slightly the above correlation and presented an empirical formula for the tubes arrangement factor F_a instead of the graphical representation by Grimison [14]. For an in-line arrangement:

$$Nu_D = 0.34F_a Re_D^{0.61} Pr^{0.31} \tag{5}$$

can be used with

$$F_a = 1 + \left(\mathscr{S}_L + \frac{7.17}{\mathscr{S}_L} - 6.52\right)\cdot\left[\frac{0.266}{(\mathscr{S}_T - 0.8)^2} - 0.12\right]\sqrt{\frac{1000}{Re_D}} \tag{6}$$

For a staggered arrangement:

$$Nu_D = 0.35F_a Re_D^{0.57} Pr^{0.31} \tag{7}$$

can be employed with

$$F_a = 1 + 0.1\mathscr{S}_L + \frac{0.34}{\mathscr{S}_T} \tag{8}$$

where $\mathscr{S}_T$ and $\mathscr{S}_L$ are the dimensionless transverse and longitudinal pitches, respectively. Žukauskas [10] gave the following experimental correlation for the average Nusselt number for a tube bank consisting of 16 or more rows:

$$Nu_D = FCRe_D^n Pr^m \tag{9}$$

where the coefficients C, m, n and the parameter F are given in tabular form ([17]). Launder and Massey [19], Fujii and Fujii [20], Dhaubhadel et al. [21], Wung and Chen [22], Murray [23] presented numerical solutions of local heat transfer for the tube bank problem for a wide range of longitudinal and transverse pitches, Reynolds and Prandtl numbers.

Beale [24] conducted a detailed numerical study of fluid flow and heat transfer in tube banks. Using complex function theory, he obtained a potential flow solution in the form of a power series. He presented his results in the form of skin friction, pressure drop, and heat transfer for different thermal boundary conditions. Later Beale and Spalding [25,26] extended the previous work for laminar fully-developed cross flow and heat transfer in tube-bank heat exchangers. They obtained a wide range of results for in-line square, rotated square, and equilateral triangle configurations.

Wilson and Bassiouny [27] developed a mathematical model to simulate the laminar and turbulent flow fields inside tube banks. They solved the conservation equations of mass, momentum and energy using an implicit finite volume procedure. They found that pressure drop and friction factor increased with the longitudinal pitch. They recommended the use of a longitudinal pitch ratio, $\mathscr{S}_L \leqslant 3$ to obtain the best performance and to achieve a high degree of compactness in an in-line arrangement, whereas $\mathscr{S}_L \leqslant 1.5$ was needed to reduce friction and enhance Nu_D in the staggered arrangement.

Mandhani et al. [28] solved the fluid flow and energy equations numerically to obtain detailed temperature fields and the distribution of Nusselt number on the surface of a typical cylinder in a cylinder bundle for the steady incompressible flow of Newtonian fluids. They found that the surface averaged value of Nusselt number increases with decreasing values of "porosity" and increasing values of Prandtl and Reynolds numbers. Their results were found in satisfactory agreement with previous numerical and experimental data for a single cylinder and for the tube banks.

The above literature review shows that almost all studies are experimental/numerical and no comprehensive analytical model exists for any in-line or staggered arrangement that can be used for a wide range of parameters. Empirical models of heat transfer, reported in the literature, were developed for specific values/ranges of longitudinal and transverse pitches, Reynolds and Prandtl numbers. The user cannot extrapolate these correlations over a wide range of operating conditions often found in existing heat exchangers. Correlations can lead to unrealistic predictions, discontinuities and numerical difficulties if they are used outside the range for which they were developed. In order to avoid these problems new comprehensive models are developed that can be used for the wide range of parameters discussed above.

2. Analysis

A tube bank usually consists of many rows of tubes in directions both parallel and perpendicular to the flow direction. The tubes may be arranged in in-line or staggered manner as shown in Figs. 1 and 2, respectively.

The total heat transfer rate from the tube bank depends upon the average heat transfer coefficient, the total surface area for heat transfer, and the inlet and outlet air temperatures, given by:

$$Q = h(N\pi DL)\Delta T_{lm} \tag{10}$$

where h is the average heat transfer coefficient, N is the total number of tubes and ΔT_{lm} is the log mean temperature difference and is given by

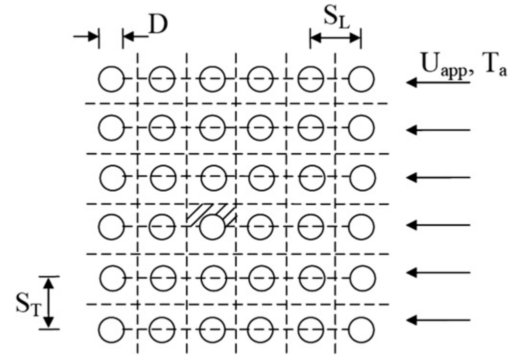

Fig. 1. Schematic of in-line arrangement.

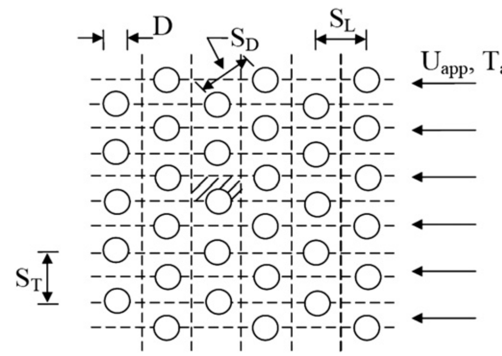

Fig. 2. Schematic of staggered arrangement.

$$\Delta T_{lm} = \frac{(T_w - T_a) - (T_w - T_o)}{\ln[(T_w - T_a)/(T_w - T_o)]} \tag{11}$$

The oulet temperature of air T_o can be determined by using energy balance and is given by

$$T_o = T_w - (T_w - T_a) \cdot \exp\left(-\frac{\pi DNh}{\rho U_{app} N_T S_T c_p}\right) \tag{12}$$

where N_T is the number of tubes in the transverse direction. The only unknown quantity in Eq. (10) is the average convection heat transfer coefficient for the tube bank that depends on the diameter of the tube, the approach velocity (U_{app}), the arrangement of tubes as well as the physical properties of the fluid (ρ, μ, c_p, k_f). The functional relationship for the average dimensionless heat transfer coefficient can be written as:

$$Nu_D = f(Re_D, Pr, \mathscr{S}_T, \mathscr{S}_L) \tag{13}$$

The conventional approach to determine Nu_D is to use experimental/numerical correlations developed by many authors (including Žukauskas and Ulinskas [10,13] and Grimison [14]) for specific values/ranges of longitudinal and transverse pitches, Reynolds and Prandtl numbers. In designing a tube bank, these correlations cannot be used due to many unknown design variables including D, N, L, and U_{app}. It is, therefore, necessary to develop analytical correlation for each arrangement that can be used for a wide range of parameters.

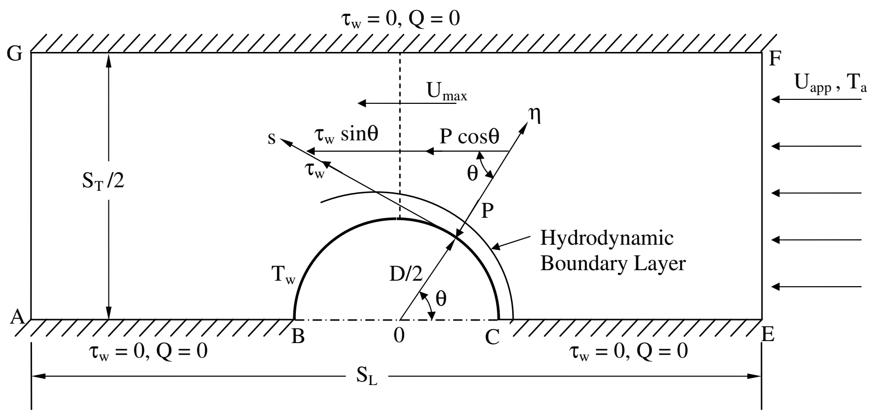

Fig. 3. Control volume for prediction of heat transfer from tube bank.

For this purpose, based on the results of previous experimental/numerical studies, a control volume (CV) is selected from the fourth row as a typical cell (Fig. 3) to study the heat transfer from an in-line or staggered tube bank. The width of the control volume is taken as unity for convenience and the length and height, in dimensionless form, are taken as $\mathscr{S}_L$ and $\mathscr{S}_T/2$ ($\equiv S_T/2D$), respectively. Because the flow is symmetrical about the horizontal center-line, the solution has been obtained for half of the flow domain, i.e., for ABCEFG in Fig. 3. The control volume surface can be regarded as impermeable, adiabatic and shear free (no mass transfer and shear work transfer across the boundary). The heat transfer between the tube and stream is Q and the wall temperature is T_w. The governing equations, velocity and temperature distributions for the CV inside the boundary layer are the same as described by Khan [29]. A fourth-order velocity profile in the hydrodynamic boundary layer and a third-order temperature profile in the thermal boundary layer are used. The potential flow velocity outside the boundary layer was obtained by using complex variable theory for both arrangements. The boundary conditions for the CV are described in Section 2.1.

2.1. Boundary conditions

The following boundary conditions are specified for the control volume of Fig. 3:

1. On the curved surfaces of the tube

   $u = 0 \quad v = 0 \quad \text{and} \quad T = T_w$

2. Along the top and bottom of the control volume

   $v = 0 \quad \tau_w = 0 \quad \text{and} \quad Q = 0$

3. At entrance of the CV, it is assumed that

   $u = U_{app} \quad \text{and} \quad T = T_a$

In reality, the velocity U_{app} is not uniform at the EF plane for any arrangement. This velocity profile depends on many factors including Reynolds number, longitudinal and transverse pitches as well as the method of tubes arrangement. These velocity profiles are presented in graphical form by Žukauskas and Ulinskas [13] for some specific cases and, therefore, it is not possible to reproduce graphical information in terms of the above mentioned factors. In order to proceed analytically, it is, therefore, assumed that the approach velocity is uniform. Due to this assumption, higher (around 12%) local heat transfer coefficients are obtained than the experimental/numerical values. In averaging the heat transfer coefficients over the entire surface, the mean values of heat transfer coefficients are reduced to some extent.

2.2. Reference velocity

The mean velocity in the minimum free cross section of the CV, U_{max}, is used as a reference velocity in the calculations of fluid flow and heat transfer for both types of arrangements, and is given by:

$$U_{max} = \max\left(\frac{\mathscr{S}_T}{\mathscr{S}_T - 1}U_{app}, \frac{\mathscr{S}_T}{\mathscr{S}_D - 1}U_{app}\right) \tag{14}$$

where $\mathscr{S}_D = \sqrt{\mathscr{S}_L^2 + (\mathscr{S}_T/2)^2}$ is the dimensionless diagonal pitch in the case of the staggered arrangement. Depending upon the maximum velocity, the Reynolds number for any arrangement can be defined as:

$$Re_D = \frac{DU_{max}}{\nu} \tag{15}$$

2.3. Heat transfer

Assuming the presence of a thin thermal boundary layer δ_T along the tube surface in the CV, the energy integral equation for the isothermal boundary condition can be written as:

$$\frac{d}{ds}\int_0^{\delta_T}(T - T_a)u\,d\eta = -\alpha\frac{\partial T}{\partial \eta}\bigg|_{\eta=0} \tag{16}$$

Using a third-order velocity profile and a fourth-order temperature profile that satisfy all the boundary conditions

([29]) and assuming $\zeta = \delta_T/\delta < 1$, Eq. (16) can be integrated to give

$$\delta_T \frac{d}{ds}[U(s)\delta_T\zeta(\lambda+12)] = 90\alpha \tag{17}$$

where $U(s)$ is the potential flow velocity outside the boundary layer, and λ is the pressure gradient parameter given by:

$$\lambda = \frac{\delta^2}{\nu}\frac{dU(s)}{ds} \tag{18}$$

The potential flow velocity outside the boundary layer was obtained by using complex variable theory and following Suh et al. [30] it can be written as:

$$U(s) = U_{\max} f(\theta) \tag{19}$$

where

$$f(\theta) = \sin\theta - 2\sin^2\left(\frac{\pi}{2a}\right) \times \left\{ \frac{\cosh\left(\frac{\pi}{a}\sin\theta\right)\sin\theta}{\cosh\left(\frac{\pi}{a}\sin\theta\right) - \cos\left(\frac{\pi}{a}\cos\theta\right)} + \sinh\left(\frac{\pi}{a}\sin\theta\right) \times \frac{\sinh\left(\frac{\pi}{a}\sin\theta\right)\sin\theta + \cos\theta\sin\left(\frac{\pi}{a}\cos\theta\right)}{\left[\cosh\left(\frac{\pi}{a}\sin\theta\right) - \cos\left(\frac{\pi}{a}\cos\theta\right)\right]^2} \right\} \tag{20}$$

for an in-line arrangement and

$$f(\theta) = \sin\theta - 2\sin^2\left(\frac{\pi}{4a}\right) \times \left\{ \frac{\cosh\left(\frac{\pi\sin\theta}{2a}\right)\sin\theta}{\cosh\left(\frac{\pi\sin\theta}{2a}\right) - \cos\left(\frac{\pi\cos\theta}{2a}\right)} - \sinh\left(\frac{\pi\sin\theta}{2a}\right) \times \frac{\sinh\left(\frac{\pi\sin\theta}{2a}\right)\sin\theta + \sin\left(\frac{\pi\cos\theta}{2a}\right)\cos\theta}{\left[\cosh\left(\frac{\pi\sin\theta}{2a}\right) - \cos\left(\frac{\pi\cos\theta}{2a}\right)\right]^2} + \frac{\cosh\left(\pi\frac{\sin\theta-2b}{2a}\right)\sin\theta}{\cosh\left(\pi\frac{\sin\theta-2b}{2a}\right) - \cos\left(\pi\frac{\cos\theta-2a}{2a}\right)} - \sinh\left(\pi\frac{\sin\theta-2b}{2a}\right) \frac{\sinh\left(\pi\frac{\sin\theta-2b}{2a}\right)\sin\theta + \sin\left(\pi\frac{\cos\theta-2a}{2a}\right)\cos\theta}{\left[\cosh\left(\pi\frac{\sin\theta-2b}{2a}\right) - \cos\left(\pi\frac{\cos\theta-2a}{2a}\right)\right]^2} \right\} \tag{21}$$

for the staggered arrangement.

Khan [29] solved Eq. (17) by using MAPLE 9 [30] and obtained the local dimensionless momentum and thermal boundary layer thicknesses. Using the definition of local and then average heat transfer coefficients, Khan [29] determined the coefficients C_1 in terms of pitch ratios for different arrangements and then correlated them to obtain single expressions in terms of longitudinal and transverse pitch ratios for both in-line and staggered arrangements. For isothermal boundary condition, these coefficients are given by:

$$C_1 = \begin{cases} [0.25 + \exp(-0.55\mathscr{S}_L)]\mathscr{S}_T^{0.285}\mathscr{S}_L^{0.212} & \text{for in-line arrangement} \\ \frac{0.61\mathscr{S}_T^{0.091}\mathscr{S}_L^{0.053}}{[1-2\exp(-1.09\mathscr{S}_L)]} & \text{for staggered arrangement} \end{cases} \tag{22}$$

Eq. (22) is valid for $1.05 \leqslant \mathscr{S}_L \leqslant 3$ and $1.05 \leqslant \mathscr{S}_T \leqslant 3$. The dimensionless heat transfer coefficient in terms of C_1, Re_D and Pr numbers can be written as:

$$Nu_D = \frac{hD}{k_f} = C_1 Re_D^{1/2} Pr^{1/3} \tag{23}$$

3. Results and discussion

According to Žukauskas and Ulinskas [13], tube banks with $\mathscr{S}_T \times \mathscr{S}_L \leqslant 1.25 \times 1.25$ are considered compact, and with $\mathscr{S}_T \times \mathscr{S}_L \geqslant 2 \times 2$ they are said to be widely spaced. For both compact and wide tube banks, Incropera and DeWitt [31] solved problem of staggered tube bank that is used for space heating. In this study, that problem is chosen for comparing the results of present analysis. In that problem, they assumed steady state conditions, negligible radiation effects, and negligible effect of change in air temperature on air properties. They used the following data to calculate air-side convection coefficient and heat rate (Table 1).

Incropera and DeWitt [31] solved their problem by using Žukauskas [10] corelations whereas the present analysis uses an analytical model. The results are shown in Table 2 for a compact bank and in Table 3 for a widely spaced bank. Table 2 shows that the present analysis gives higher results (around 22%) than Incropera and DeWitt [31]. This discrepancy is due to the fact that Žukauskas [10] gave only one correlation for $S_T/S_L < 2$. In both cases (compact and widely spaced), Incropera and DeWitt [31] used the same correlation for heat transfer. Furthermore, Žukauskas [10] correlation for compact banks depends on the incoming velocity rather than the maximum velocity in the minimum

Table 1
Data used by Incropera and DeWitt [31] for staggered tube bank

Quantity	Dimension
Tube diameter (mm)	16.4
Longitudinal pitch (mm)	20.5, 34.3
Transverse pitch (mm)	20.5, 31.3
Number of tubes (Staggered)	8 × 7
Approach velocity (m/s)	6
Thermal conductivity of air (W/m K)	0.0253
Density of air (kg/m^3)	1.217
Specific heat of air (J/kg K)	1007
Kinematic viscosity (m^2/s)	14.82×10^{-6}
Prandtl number (Air)	0.701
Ambient temperature (°C)	15
Tube surface temperature (°C)	70

Table 2
Comparison of results for compact tube bank (1.25 × 1.25)

	NuD	h (W/m^2 K)	T_o (°C)	Q (kW)
Incropera and DeWitt [31]	152.0	234.0	38.5	28.4
Present analysis	196.1	302.5	43.3	34.1

Table 3
Comparison of results for wide tube bank (2.1 × 2.1)

	Nu_D	h (W/m^2 K)	T_o (°C)	Q (kW)
Incropera and DeWitt [31]	87.9	135.6	25.5	19.4
Present analysis	88.3	136.2	26.9	19.2

flow area. For wide tube banks, Table 3 shows good agreement. The comparison of Tables 2 and 3 shows that heat transfer decreases with increasing pitch ratio.

The effects of longitudinal and transverse pitch ratios on heat transfer for both arrangements are shown in Figs. 4 and 5. In both arrangements, the heat transfer increases mainly with decreasing longitudinal pith ratio, and to a lesser extent with increasing transverse pitch ratio. Compact banks (in-line or staggered) indicate higher heat transfer rates than widely spaced ones. For the same pitch ratio, the heat transfer is higher in a staggered bank than in an in-line bank. This is due to the fact that in a staggered bank the path of the main flow is more complicated and a greater portion of the surface area of downstream tubes remains in this path.

The comparison of heat transfer coefficients for compact and widely spaced banks is shown in Figs. 6 and 7 for both arrangements. The heat transfer increases linearly with Reynolds number on the logarithmic scale. It is observed that in both banks staggered arrangement gives higher heat transfer than in-line arrangement and this difference diminishes as the the pitch ratio increases. For larger pitch ratios, arrangement of the tubes makes no difference with reference to heat transfer.

Average heat transfer values for a compact in-line bank are plotted versus Re_D in Fig. 8. On the logarthimic scale, heat transfer values increase linearly with the Reynolds numbers. The present values are compared with Grimison [14], Žukauskas [10], and Žukauskas and Ulinskas [13]. Grimison [14] correlation is in good agreement with the analytical results but Žukauskas and Ulinskas [13] experimental data differs considerably from the present results. This is due to the fact that in compact in-line banks, the incoming flow increases significantly near the minimum cross-sectional area that influences only a small part of the tube surface. For such banks, they evaluated their correlations at the incoming velocity rather than at the maximum velocity in the minimum flow area.

The variation of average heat transfer coefficient versus Reynolds number is shown in Fig. 9 for 2×2 asymmetrical

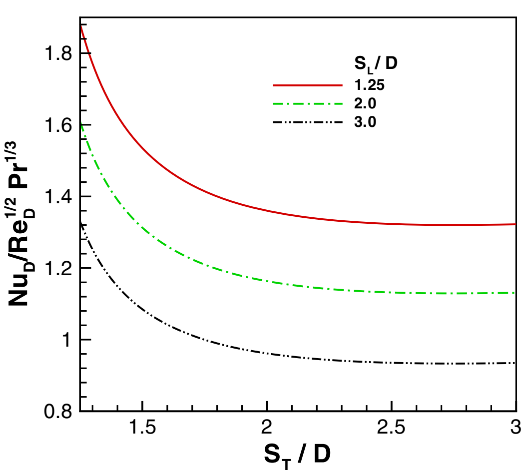

Fig. 4. Variation of heat transfer parameter with transverse pitch for in-line arrangement.

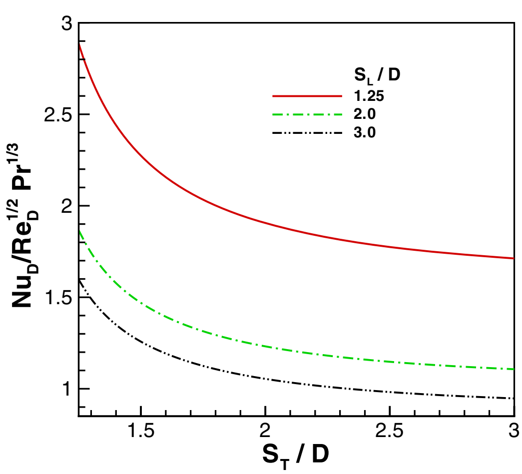

Fig. 5. Variation of heat transfer parameter with transverse pitch for staggered arrangement.

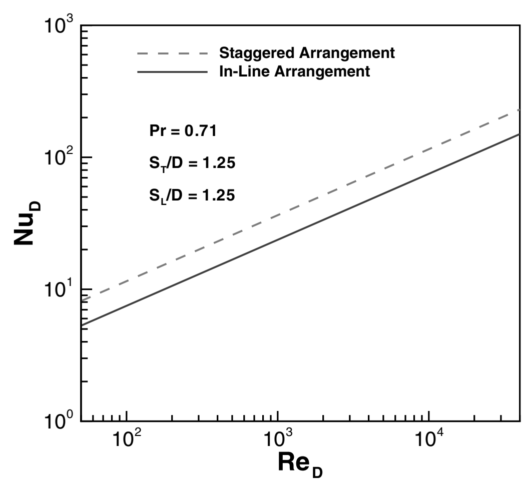

Fig. 6. Comparison of both arrangements for 1.25×1.25.

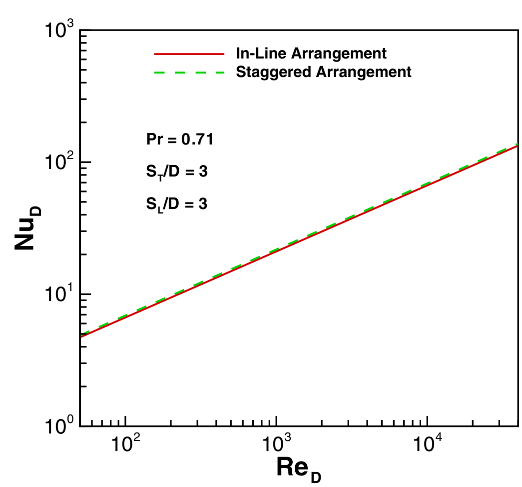

Fig. 7. Comparison of both arrangements for 3×3.

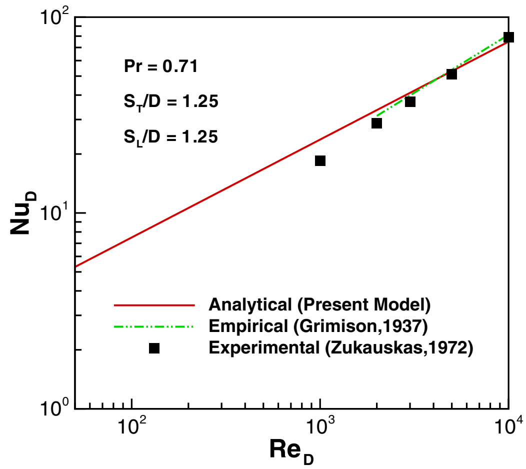

Fig. 8. Average Nusselt numbers for in-line arrangement 1.25 × 1.25.

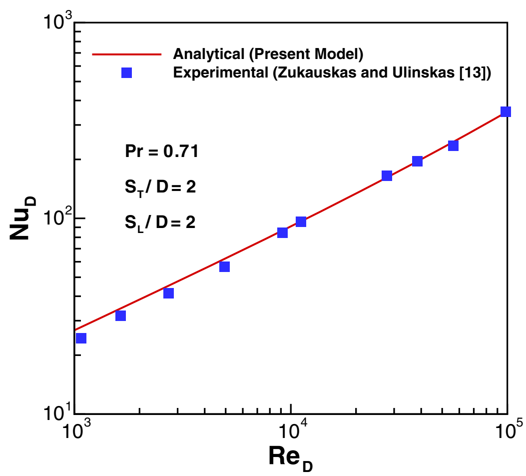

Fig. 9. Average Nusselt numbers for staggered arrangement 2 × 2.

staggered tube bank. For the sake of comparison, empirical correlations of Grimison [14] and Hausen [18] as well as experimental data of Žukauskas and Ulinskas [13] are shown on the same figure. The present results are in good agreement with the availabe data in the laminar flow range.

4. Conclusions

Heat transfer from tube banks in crossflow is investigated analytically and simplified models of heat transfer for both arrangements (in-line and staggered) are presented. The coefficient C_1 in Eq. (19) is determined by fitting the analytical results obtained for various pitch ratios in both arrangements. The results obtained from this investigation are as follows:

1. Both models can be applied over a wide range of parameters and are suitable for use in the design of tube banks.
2. The average heat transfer coefficients for tube banks in crossflow depend on the longitudinal and transverse pitches, Reynolds and Prandtl numbers.
3. Compact banks (in-line or staggered) indicate higher heat transfer rates than widely spaced ones.
4. The staggered arrangement gives higher heat transfer rates than the in-line arrangement.

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